Performance Testing For Database Systems

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BY
Anupam Sanghi

Computer Science and Automation
Indian Institute of Science
Bangalore – 560 012 (INDIA)

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Declaration of Originality

I, Anupam Sanghi, with SR No. 04-04-00-10-41-14-1-11143 hereby declare that the material presented in the thesis titled

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In my capacity as supervisor of the above-mentioned work, I certify that the above statements are true to the best of my knowledge, and I have carried out due diligence to ensure the originality of the report.

Advisor Name: Prof. Jayant R. Haritsa

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My Family and Friends
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Abstract

Database systems’ testing, in the state of the art uses synthetic data and workload generators such as TPC-H and TPC-DS. Synthetic generators are used because the real customer data is hard to obtain due to its sensitive nature as well as huge size. But these synthetic benchmarks usually have a fixed schema and workload, and also they do not provide much flexibility in data generation except for the size of the database. Due to these limitations, they often fail to capture realistic scenarios.

In this project, we have constructed a workload-dependent synthetic data generator, which ensures that the query performance is similar on the real and simulated environments for a predefined workload. By similar query performance, we mean that the executor does similar volumetric processing of data for each node in the execution plan tree. The special features of the generation algorithm are that it is independent of the size of the database and has the capability of generating and supplying data on-the-fly. Hence, the time and space overheads get eliminated, which would be extremely useful in futuristic big data scenarios, where these features are indispensable. Since, the generator does not require any static data storage, it subscribes to the philosophy of CODD (COnstructing Dataless Databases), a tool that helps to do compile-time system testing by constructing dataless databases.
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Chapter 1

Introduction

Effective performance testing of database engines and applications is predicated on the ability to easily construct alternative scenarios with regard to the database contents [16]. In the state of the art, it involves executing a set of queries (known as query workload) on synthetic databases. A query is processed in two phases - at compile-time, the query optimizer picks an optimal plan from a set of plans. Thereafter, the executor runs the chosen plan on the database, also called as execution-time processing, to give the result. Therefore, testing can be broadly categorized into compile-time testing and execution-time testing. There are several synthetic benchmarks like TPC-H [3], TPC-DS [2] that are commonly used to carry out performance tests in various domains. But unfortunately, these benchmarks are far from realistic customer scenarios. Due to its sensitive nature, the customer data is also hard to obtain. This therefore results in unidentified bugs before deployment [15].

1.1 Synthetic Benchmarks: Shortcomings

The major limitations of techniques that use synthetic benchmarks are as follows:

- **Fixed schema and workload:** The synthetic data generators possess a fixed schema and workload, which fail to accommodate various realistic settings.

- **Inability to tune the generated data:** Not much flexibility is given in generation. The data usually has a fixed predefined distribution, which might again fail to adapt to the real scenarios.

- **Infeasible at big data scale:** In today’s era of so-called big data systems, the data is of prohibitively large sizes. It is easy to see that the traditional testing techniques become
completely impractical simply because of the time and space overheads that arise due to the tedious data loading and storing processes.

Motivated by the above problem, [16] proposed a tool called CODD\(^1\) (COnstructing Data-less Databases), that took the first step towards alleviating the problem of effective testing of big data systems, by implementing a new metaphor of “data-less databases”. It aims to provide a platform to construct virtual databases with no static (stored) data that can simulate the desired scenario easily. This can therefore overcome the above mentioned limitations.

1.2 Compile-time Testing

CODD currently supports compile-time testing aspect of the database systems\(^2\). At compile-time, the query optimizer uses the meta-data characteristics of the database to pick the optimal plan for execution. Thus, this stage does not require the actual data to be present in the system. CODD leverages this property by providing a platform to directly construct the metadata. Therefore, database environments with the desired metadata characteristics can be efficiently simulated without persistently generating and/or storing their contents. Correct simulation implies obtaining identical plan diagrams [13], indicating that the optimizer behaves identically on the real and simulated scenarios. Another unique feature of CODD is its support for automated scaling of meta-data instances, which can be used to mimic futuristic scenarios. For instance, consider the situation where a database engineer wishes to evaluate the query optimizer’s behaviour on a futuristic big data set-up featuring “yottabyte”\((10^{24}\) bytes\) sized relational tables. Obviously, just generating this data, let alone storing it, is practically infeasible even on the best of systems, but using CODD this can be easily modelled within a few minutes on a vanilla laptop [14].

1.3 Execution-time Testing

Unlike compile-time, execution-time testing does need the actual data to be present, at least transiently, in the system. Therefore, solving the problem for execution-time testing becomes challenging. To handle this, we used the technique of workload-dependent data generation to obtain synthetic data that can mimic the performance of the real database on a predefined workload. By mimicking, we mean that the executor does similar volumetric processing of data for each node in the execution plan tree in both real and simulated environments. Further, our

\(^1\)In archaic English, **cod** means “empty shell”, symbolising the data-less context.

\(^2\)Currently CODD supports several industrial-strength optimizers, including IBM DB2, Microsoft SQL Server, Oracle, HP SQL/MX and PostgreSQL.
generation mechanism does not depend on the size of the database, something indispensable in big data scenarios.

1.4 Applications

Volumetric execution-time testing as mentioned above has numerous applications like:

- **Component Testing:** On introducing a new component/technique into the engine, testing its performance is required. This can be done easily by using our generator to get data with the desired properties. This data can be supplied to the executor on demand, which can forward it to the newly added component.

- **Resolve customer issues:** This application was motivated from HP Enterprise. Client organizations outsource the testing of their database applications. Here, the data generator can serve as a platform that can provide synthetic data possessing the desired properties of the client’s data. This would eliminate the requirement of getting access to the client’s production system and yet can resolve the performance related problems of the engine. For example, solutions might be (a) propose a hardware configuration, (b) alternate algorithm for an operator for efficient execution.

- **Testing on Futuristic Scenarios:** If the organisations want to perform testing on futuristic scenarios, where the data size is scaled-up, then this can also be done easily by scaling the inputs given to CODD and it will ensure that the generated metadata and synthetic data mimics the desired scenario.

1.5 Prior Work

There are various techniques in the literature that deal with synthetic data generation. Some techniques like [7, 8] focus on making the generation process parallelizable and scalable for pre-defined data distributions. But, these fail to resemble the real scenarios as they do not capture the correlations that exist between the attributes within and across tables.

Another line of work deals with generating data from the statistical metadata [15], but this also loses the non-trivial correlations and therefore fail to mimic the real scenarios.

QAGen is a tool proposed in [6, 9], where the idea of using annotated query plan trees was introduced. This was further extended in a tool called MyBenchmark [10, 11]. They are based on the approach called symbolic query processing. It constructs a symbolic database\(^1\) on a per query basis, to which constraints are added depending on the input plans and finally these

\(^1\)A symbolic database is like a template database, where the attribute values are variables and not constants.
symbolic databases are integrated and instantiated by using constraint satisfaction program. Though this technique is able to handle a larger set of queries, it defeats our purpose of making an on-the-fly data generator since it would require $N$ symbolic databases for $N$ input trees in the very first step of the algorithm.

Arasu et. al. [4, 5] proposed a technique that generates the data using cardinality constraints. In this technique we form linear programs (LPs) using the cardinality constraints and use the solution to generate the data that tries to satisfy all the constraints in expectation. Our data generator is based on this work. We have also addressed some of the limitations that we will discuss next.

1.6 Our Contributions

We have modified the work done in [4] in order to overcome some of its limitations. The modifications are as follows:

- **Support on-the-fly data generation**: In the earlier technique, a view is created corresponding to every relation and an LP is formulated for each view separately. Thereafter, from the LP solution, the views are instantiated and relations are constructed from them. The modified technique does not require instantiating the views, rather we create relation summaries. These summaries (much smaller in size than the actual relations) are capable of supporting on-the-fly generation.

- **Handle richer class of schemas**: The earlier technique handled only snowflake schemas. We have extended it to support any schema with non-cyclic dependencies between the relations. The snowflake schema is a special category of non-cyclic dependencies, where the dependency graph is a tree.

- **Reduce the error in satisfying the constraints**: The earlier approach used sampling for instantiation which induced some errors in satisfying the cardinality constraints. We have made the generation algorithm deterministic, which ensures that no sampling errors get added up.

- **Accept small-scale generic projections**: The earlier work did not handle generic projection operators. They were limited to the cases where the projections are non-overlapping. We have proposed a new technique that can handle any kind of projection operators.

Our generator takes the database schema and a set of annotated logical query plans (ALQPs) as the input. From the ALQPs set, we generate a cardinality constraint for each operator in the
plan tree. Further, these constraints are converted into a linear program, which then is passed on to the LP solver. The result from the LP solver is finally fed into the relations generator that has the capability of supplying tuples on demand.

We verified our work on TPC-DS benchmark, where we constructed 14 queries by modifying the original queries to suit the assumptions that the algorithm requires. In our results, we found that the query performance, with regard to the volumetric processing at each node of the plan tree, was very similar on the real 10 GB TPC-DS database and our synthetically generated one.

1.7 Roadmap

In the rest of this document, we shall first discuss the problem framework in Chapter 2 including the premilinaries, problem definition and assumptions. Further, Chapter 3 discusses the architecture of the data generator along with a detailed description of its various components. Thereafter, Chapter 4 includes an empirical evaluation of our generator and finally we conclude with various open problems that can be explored.
Chapter 2

Problem Framework

2.1 Preliminaries

2.1.1 Logical Query Plan (LQP)

A logical query plan is an extended relational algebra tree. It gives the set of operations involved in executing a query along with their sequence. For example, consider the following query on the TPC-DS database:

\[
\begin{align*}
\text{SELECT} & \quad (*) \\
\text{FROM} & \quad \text{Web\_sales} \ W, \ \text{Date\_dim} \ D, \ \text{Item} \ I \\
\text{WHERE} & \quad D.d\text{-month\_seq} \geq 1211 \\
& \quad \text{AND} \ W.ws\_sold\_date\_sk = D.d\_date\_sk \\
& \quad \text{AND} \ W.ws\_item\_sk = I.i\_item\_sk
\end{align*}
\]

For the above query, a possible LQP is shown in Figure 2.1.

2.1.2 Annotated Logical Query Plan (ALQP)

An ALQP is an LQP, which in addition also gives input and output cardinalities for each operator in the tree. The input cardinality specifies the number of tuples that reach the operator and the output cardinality gives the number of tuples that leave the operator. For example, for the above LQP, a possible ALQP is the shown in Figure 2.2.
2.1.3 Cardinality Constraints

Assuming that \( R_1, ..., R_l \) are the set of relations in the database, each operator in the ALQP corresponds to a cardinality constraint that can be written in the following form:

\[
|\pi_A \sigma_P (R_{i_1} \bowtie ... \bowtie R_{i_p})| = k
\]
where $A$ denotes a set of attributes, $P$ is a selection predicate, and $k$ is a non-negative integer.

For example, the constraints corresponding to the ALQP shown in Figure 2.2 are:

$$|W| = 7197559 \quad (2.1)$$

$$|D| = 73066 \quad (2.2)$$

$$|I| = 102006 \quad (2.3)$$

$$|\sigma_{d_{\text{month}, \text{seq} > 1211}}(D)| = 36192 \quad (2.4)$$

$$|\sigma_{d_{\text{month}, \text{seq} > 1211}}(W \bowtie D)| = 3131870 \quad (2.5)$$

$$|\sigma_{d_{\text{month}, \text{seq} > 1211}}(W \bowtie D \bowtie I)| = 3131870 \quad (2.6)$$

2.2 Problem

2.2.1 Statement

We shall now formally state the problem:

Given a database schema $S$ and a set of ALQPs $W$, generate a database instance that conforms to $S$ and satisfies all the cardinality constraints ($C_1, C_2, ..., C_m$) generated from $W$.

In the decision version 1 of this problem is $NEXP$-complete [4].

2.2.2 Assumptions

The assumptions made in our work are:

- All the joins appearing in the constraints are primary key-foreign key joins.
- The dependency (due to joins) between relations should be non-cyclic.
- Selection predicates include only non-key attributes.

1 In the decision version of the problem, the output is YES if there exists a database instance that satisfies all the constraints and NO, otherwise.

8
Chapter 3

Data Generation

Our data generator takes the schema of the desired database as the input. Along with it, the set of ALQPs are also given as the input. The ALQPs are generated after executing the queries on the original database. These ALQPs can be easily fetched from the execution-plan information that the database engines provide. The generator uses these inputs to give the synthetic database (on-the-fly) as the output. We next describe the architecture of the generator.

3.1 Architecture

Figure 3.1 provides an overview of the architecture of the data generator. Its various components are:

- **Parser**: The purpose of parser is to take \( W \) as input and give the set of cardinality constraints as described in Section 2.1.3. Note that for now we are not considering projection operator. Therefore, all the constraints will be of the form:

  \[
  |\sigma_{\mathcal{P}}(R_{i_1} \bowtie ... \bowtie R_{i_p})| = k
  \]  

- **View Generator**: This component takes the database schema as input and creates a view \( V_i \) corresponding to every relation \( R_i \). Creation of views help us to get rid of the join expressions in the constraints, i.e., once the views are created, each constraint can be re-written as a selection predicate over a single view only. We shall see this component in detail in Section 3.2.1.

- **LP Formulator**: This component uses the views given by the view generator to re-write the cardinality constraints given by the parser. Further, an LP is created for each view.
This is done by converting each constraint (on that view) to a corresponding equation. Once this is done, the system of equations is passed on to the LP solver. We shall discuss the details of constructing equations in detail in Section 3.2.2.

- **LP Solver**: This component takes the system of equations and gives one of the feasible solutions\(^1\). We use the GNU Linear Programming Kit (GLPK) [1] for solving the LP.

- **Relation Generator**: This component takes the solution given by the LP solver and (i) makes it consistent across the views, (ii) constructs compressed relations from it, which is sufficient to generate the entire relation. Section 3.2.3 discusses this component in detail.

- **Tuple Generator**: The compressed relations that we obtain from the relations generator serve as the seeds to the tuple generator. The tuple generator has the capability of generating a tuple(s) on demand for any relation \(R_i\). The detailed description of this component is mentioned in Section 3.2.4.

\(^{1}\)The system of equations can have infinite solutions. The solution corresponding to the original database instance might differ from the solution we get here. But, both the solutions would satisfy all the constraints.
3.2 Details

3.2.1 View Generator

As discussed earlier, the purpose of this component is to simplify the constraints such that we can replace all the join expressions by a single view. For this we need to construct a view $V_i$ corresponding to each relation $R_i$. A view $V_i$ can be considered as a set of non-key attributes that are present in either $R_i$ or in any other relation on which $R_i$ depends. The dependencies between relation can be seen from the dependency graph.

Dependency Graph: In a dependency graph, we create a node for every relation. A directed edge from a node $R_i$ to $R_j$ is added, if $R_i$ contains a foreign-key referencing $R_j$. Now, a relation $R_i$ is said to be dependent on relation $R_j$ if there exists a path from $R_i$ to $R_j$ in the dependency graph.

Let us see the following example: Consider a database having following four relations:

- \( Catalog\_sales(PK_1, FK_C, FK_D, cs\_sales\_price) \)
- \( Date\_dim(PK_3, d\_qoy, d\_year) \)
- \( Customer(PK_2, FK\_CA) \)
- \( Customer\_address(PK_4, ca\_state) \)

Here, $PK_1$, $PK_2$, $PK_3$, $PK_4$ are the primary keys of the respective relations. $FK_C$ references to $Customer$, $FK_D$ references to $Date\_dim$ and $FK\_CA$ references to $Customer\_address$. Therefore, the dependency graph would be as shown in Figure 3.2.
After executing the View Generation Algorithm (shown in Algorithm 1), the views thus obtained would be:

- \( \text{Catalog\_sales}'(\text{ca\_state, } d\_qoy, d\_year, cs\_sales\_price) \)
- \( \text{Date\_dim}'(d\_qoy, d\_year) \)
- \( \text{Customer}'(\text{ca\_state}) \)
- \( \text{Customer\_address}'(\text{ca\_state}) \)

**Algorithm 1: View Generation Algorithm**

**Input:** \( \{R_i\}_{i\in[n]} \) \hspace{1cm} \( \triangleright \) Set of relations \( R_1, \ldots, R_n \)

**Output:** \( \{V_i\}_{i\in[n]} \) \hspace{1cm} \( \triangleright \) Set of views \( V_1, \ldots, V_n \)

**Functionalities Used:**
- \( \text{getFK}(R_i) \) \hspace{1cm} \( \triangleright \) Returns set of \( <\text{attr, ref}> \) pairs corresponding to the FKs in \( R_i \)
- \( \text{getNonKey}(R_i) \) \hspace{1cm} \( \triangleright \) Returns set of non-key attributes in \( R_i \)

1: \( \text{procedure getView}(R_i, \{V_i\}_{i\in[n]}, \{\text{flag}_i\}_{i\in[n]}) \) \hspace{1cm} \( \triangleright \) The function returns the view \( V_i \) for a relation \( R_i \)
2: \( \quad \text{if} ! (\text{flag}_i) \) then
3: \( \quad \quad \text{V}_i \leftarrow \text{getNonKey}(R_i) \) \hspace{1cm} \( \triangleright \) The non-key attributes of \( R_i \) are inserted in \( V_i \)
4: \( \quad \quad \text{for} < \text{attr, ref}> \text{ in getFK}(R_i) \text{ do} \)
5: \( \quad \quad \quad \text{V}_i \leftarrow \text{V}_i \cup \text{getView}(\text{ref}, \{\text{V}_i\}_{i\in[n]}, \{\text{flag}_i\}_{i\in[n]}) \) \hspace{1cm} \( \triangleright \) Call the function iteratively for reference table corresponding to every foreign key in \( R_i \)
6: \( \quad \quad \text{end for} \)
7: \( \quad \text{flag}_i \leftarrow \text{true} \)
8: \( \quad \text{end if} \)
9: \( \text{return V}_i \)
10: \( \text{end procedure} \)
11: \( \textbf{Init:} \ V_i \leftarrow \emptyset, \text{flag}_i \leftarrow \text{false} \ \forall i \in [n] \) \hspace{1cm} \( \triangleright \) Initialize each view with an empty set. \( \text{flag}_i \) is used to check if \( V_i \) has been generated
12: \( \text{for } i \leftarrow 1 \text{ to } n \text{ do} \)
13: \( \quad \text{V}_i \leftarrow \text{getView}(R_i, \{\text{V}_i\}_{i\in[n]}, \{\text{flag}_i\}_{i\in[n]}) \)
14: \( \text{end for} \)
3.2.2 LP Formulator

The LP Formulator receives the set of constraints from the parser and the set of views from the view generator. The first task it does is to re-write the constraints by replacing the join expressions with appropriate views. Say we have a constraint having join: \( R_i \bowtie R_k \). Since, we have assumed that all joins are of primary key-foreign key type, one of these is a dependent relation. Say \( R_i \) depends on \( R_k \). In such a case, we shall replace the expression \( R_i \bowtie R_k \) with \( V_i \). Because of the way we constructed the views, it is easy to see that we can apply the predicate that was there on \( R_i \bowtie R_k \) to \( V_i \). Likewise, all the join expressions can be expressed in terms of a single view respectively.

Continuing on the example as in Figure 2.2, considering that the dependency graph is rooted at node \( W \) having two children \( D \) and \( I \), the cardinality constraints 2.5 and 2.6 can be rewritten as:

\[
|\sigma_{d_{\text{month_seq}}=1211}(W')| = 3131870 \quad (3.2)
\]

\[
|\sigma_{d_{\text{month_seq}}=1211}(W')| = 3131870 \quad (3.3)
\]

where \( W' \) represents the view corresponding to \( W \).

So now we can solve for each view separately. For each view \( V_i \), we will find the set of constraints imposed on \( V_i \). A constraint \( C_j \) on \( V_i \) will be of the following form:

\[
|\sigma_{p_j}(V_i)| = k_j \quad (3.4)
\]

Now, let us assume we have a single view \( V \) having a set of attributes \( A_1, A_2, ..., A_n \). We need to formulate an LP for the view \( V \). Let the domain of an attribute \( A_i \) be represented by \( \text{Dom}(A_i) \). We assume that the domain of all the attributes are positive integers bounded by an integer \( D \). For attributes with non-integral domains, we can map the values to integers. This assumption is to simplify the analysis and can be removed easily.

Say we are given a set of \( m \) constraints that \( V \) satisfies. Each constraint \( C_j \) (1 \( \leq j \leq m \)) for simplicity can be expressed as: \( <p_j, k_j> \), which means that the number of tuples (rows) satisfying the condition \( p_j \) is equal to \( k_j \). For every tuple \( t \in \text{Dom}(A_1) \times \text{Dom}(A_2) \times ... \times \text{Dom}(A_n) \), we create a variable \( x_t \) representing the number of copies of \( t \) in \( V \). Now, for each of the \( m \) constraints \( C_j \) (1 \( \leq j \leq m \)), we create a linear equation:

\[
\sum_{t: p_j(t) = \text{true}} x_t = k_j
\]
In addition, we also require that $x_t \geq 0 \ \forall t$ as the number of tuples are always non-negative integers. Since, the solution of the LP need not be integral, we shall use simple rounding technique.

Further, since the number of variables in the LP is proportional to the domain size, which can be huge, there are some optimizations that can be done to reduce the size of the LP. We will now discuss these optimizations.

3.2.2.1 Domain Decomposition

A set $v^i$ is created for each attribute $A_i$. Values in $v^i$ are added according to the following: We iterate over the constraints and if a constraint has

- $A_i > a$ or $A_i < a$, we add $a$ in $v^i$.
- $A_i > a$ or $A_i <= a$, we add $a + 1$ in $v^i$.
- $A_i = a$, we add $a$ and $a + 1$ both in $v^i$.

All the other constraints can be expressed as combinations of the above. In addition, we also add 1 (minimum value in domain) in $v^i$ if not already present. Let $v_1^i, v_2^i, ..., v_l^i$ represent the constants (in increasing order) in the set $v^i$. Now we can divide the domain of an attribute $A_i$ into a set of $l_i$ intervals $I^i : [v_q^i, v_{q+1}^i)$ $(1 \leq q < l_i) \cup [v_{l_i}^i, )$. The semantics of the variables can now be modified such that we introduce a variable $x_{t'}$ for each interval combination $t' \in I^1 \times I^2 \times ... \times I^n$, representing the number of tuples lying in the interval combination $t'$.

Therefore, now for each constraint $C_j (1 \leq j \leq m)$, the linear equation would be:

$$\sum_{t':P_j(t')=true} x_{t'} = k_j$$

Here as well we will have the additional constraint of $x_{t'} > 0 \ \forall t'$.

Example: Let us see the LP formulation for relation `Catalog_sales` having following constraints:

$$|CS| = 14401261$$

(3.5)

$$|\sigma_{cs\_sales\_price>150}(CS)| = 734606$$

(3.6)

$$|\sigma_{cs\_sales\_price>150 \land ca\_state=1}(CS)| = 13806$$

(3.7)
These constraints can be converted into the corresponding LP equations as follows:

\[ x_{[1,151]}(1) + x_{[1,151]}(2) + x_{[151]}(1,2) + x_{[151]}(2) = 14401261 \]  
\[ (3.8) \]

\[ x_{[151]}(1,2) + x_{[151]}(2) = 734606 \]  
\[ (3.9) \]

\[ x_{[151]}(1,2) = 13806 \]  
\[ (3.10) \]

Therefore, we can see that by applying this optimization, the number of variables are a function of the size of the sets \( v_i \)'s instead of the domain size. But, even after applying this optimization, the number of variables are exponential in the number of attributes. To further reduce the LP's complexity, we will next look at another optimization that tries to decompose the view into smaller views.

### 3.2.2.2 View Decomposition

In this optimization, we decompose a view into small components having fewer attributes. The constraints are then applied to these small components. Since, the number of attributes are fewer, the size of the LP also gets reduced.

The algorithm consists of the following steps:

- Constructing a graph from the constraints.
- Identifying the smaller components.
- Applying constraints on the smaller components.

**Graph Construction:** Construct a graph \( G = (V, E) \), where vertices corresponds to the attributes in the view \( V \) and we add an edge \((A_i, A_j)\) in \( G \) if there exists a constraint \( C \) in which attributes \( A_i \) and \( A_j \) co-appear. We also add more edges to the graph in order to make it *chordal*. A graph is chordal if each cycle of length 4 or more has a *chord*; a chord is an edge joining two non-adjacent nodes of a cycle. It is easy to see that a chord joining vertices corresponding to attributes \( A_i \) and \( A_j \) can always be added by assuming that the original set of constraints had the trivial constraint:

\[ |\sigma_{A_i \geq 1} \land A_j \geq 1| = |V| \]

We converted the graph to chordal because the chordal graphs have a special property [12] that allows us to construct the original view from the decomposed components.
**Identifying Smaller Components:** These smaller components are nothing but the *maximal cliques* obtained from the graph. Let $\mathcal{A}_{c_i}$ represent the set of attributes that clique $c_i$ contains.

**Adding Constraints:** Instead of applying constraints to the complete view, we shall now apply the constraints to the cliques. For each clique, we will add all the constraints that are within their scope. In addition, since cliques can have common attributes as well, we need to add more constraints to ensure consistency across cliques. Consider two sets of attributes $\mathcal{A}_{c_i}$ and $\mathcal{A}_{c_j}$ such that $\mathcal{A}_{c_i} \cap \mathcal{A}_{c_j} \neq \emptyset$. Further for any $p \in \text{Dom}(\mathcal{A}_{c_i} \cap \mathcal{A}_{c_j})$, let $\text{Ext}_{\mathcal{A}_{c_i}}(p)$ denote the set of assignment to $\mathcal{A}_{c_i}$ that is consistent with the assignment $p$. We include the following equation for each $p \in \text{Dom}(\mathcal{A}_{c_i} \cap \mathcal{A}_{c_j})$ in the LP:

$$\sum_{y \in \text{Ext}_{\mathcal{A}_{c_i}}(p)} x_y = \sum_{z \in \text{Ext}_{\mathcal{A}_{c_j}}(p)} x_z$$

### 3.2.3 Relation Generator

The LP solver gives the result for each clique $c_i$ in the form of the set:

$$\{(x, k^i_x) | x \in \text{Dom}(\mathcal{A}_{c_i})\}$$

where $x$ represents an interval combination and $k^i_x$ represents the number of tuples in the view that lie in the interval combination $x$. Note that $x$ constitutes of intervals of those attributes that are present in $\mathcal{A}_{c_i}$.

Since, the above solution has intervals for a set of attributes, to get a value combination from this, we need to pick a value from each interval. We pick the minimum value in every interval to get the value combination. We do this to minimize the error that is incurred in the view consistency algorithm (we shall discuss this in Section 3.2.3.2)$^1$. Now on we shall use $x$ to represent the value combination thus obtained.

Let us call these solution sets as the *clique-solution sets*. We might not explicitly mention it always, but when we say $x \in \text{Dom}(\mathcal{A}_{c_i})$, we consider only those domain values for which the corresponding $k^i_x$ is non-zero since we do not need to store the domain values that do not occur from our LP solution.

The relation generator does the following:

$^1$any deterministic method of picking a value in an interval can be adopted to reduce the error
• First, it merges the clique-solution sets to obtain the solution for the complete view.

• Then, it makes the views consistent.

• Finally, it constructs relation summaries.

### 3.2.3.1 Obtaining View Solution

The solution for the complete view is obtained by merging the cliques (Algorithm 2). This is done by first ordering the cliques using the Order Cliques Procedure (Algorithm 3). This merge ordering is necessary in order to retain all the constraints that each clique solution implies. In every iteration of ordering algorithm, we look for a clique $c_i$ that is independent of the observed cliques so far except for the attributes that are common between $c_i$ and the observed cliques. Since the graph is chordal, we are guaranteed to get at least one such ordering.

Once we get the ordering, we then merge the cliques-solution sets one by one in that order. Merging is a three step process. We first sort the clique-solution sets based on their common attributes, then use the Align Procedure (Algorithm 4) to split the rows in the two clique-solution sets in such a way that the corresponding rows in the two sets have same values for the number of tuples entry. Finally we join the two sets using the Merge Procedure (Algorithm 5).

**Example:** Consider a case where we need to merge two clique-solution sets to obtain a view solution set. Let the two clique-solution sets be as follows:

<table>
<thead>
<tr>
<th>Clique-solution 1</th>
<th>Clique-solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

After sorting on common attribute attribute B, we would obtain:
### Clique-solution 1

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>number of tuples</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

### Clique-solution 2

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>number of tuples</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>100</td>
</tr>
</tbody>
</table>

After aligning the two sets, we obtain:

### Clique-solution 1

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>number of tuples</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
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<td>30</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

### Clique-solution 2

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>number of tuples</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>100</td>
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<td>50</td>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>60</td>
</tr>
</tbody>
</table>

Finally, after merging, the view solution set thus obtained is:

### View solution

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>number of tuples</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>5</td>
<td>100</td>
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<tr>
<td>30</td>
<td>50</td>
<td>15</td>
<td>60</td>
</tr>
</tbody>
</table>
Algorithm 2 Clique Merging Algorithm

**Input:** \( \{(x, k_x^i) \}_{x \in \text{Dom}(A_i)} \}_{i \in [l]} \cup \{c_i \}_{i \in [l]} \) \( \triangleright \) Set of value combinations with cardinalities for each clique and the set of cliques

**Output:** \( X_V \) \( \triangleright \) Set of value combinations with cardinalities for the view \( V \)

**Functionalities Used:**
- \( \text{sort}(X, S) \) \( \triangleright \) Sort \( X \) on value combinations corresponding to the attributes present in the set \( S \)

1: **Init:** \( V' \leftarrow \emptyset \) \( \triangleright \) \( V' \) stores the attributes that are present in the merged set of cliques; initialized with an empty set

2: \( C \leftarrow \text{ORDERCLIQUES}\{c_i \}_{i \in [l]} \) \( \triangleright \) \( C \) stores the order in which cliques are to be merged

3: \( V' \leftarrow C[0] \) \( \triangleright \) The attributes of the first clique are added to \( V' \)

4: \( X_V \leftarrow \{(t, k_t)\}_{t \in \text{Dom}(V')} \) \( \triangleright \) \( X_V \) stores the set of value combinations with their respective cardinalities for the attributes present in \( V' \)

5: **for** \( j \leftarrow 1 \) to \( l - 1 \) **do**

6: \( X_{C_j} \leftarrow \{(x, k_x)\}_{x \in \text{Dom}(C_j)} \) \( \triangleright \) \( X_{C_j} \) stores the set of value combinations with their respective cardinalities for the attributes present in the cluster \( C_j \)

7: \( X_{C_j} \leftarrow \text{sort}(X_{C_j}, V' \cap C[j]) \) \( \triangleright \) Sort value combinations in \( X_{C_j} \) on the attributes that are common with \( V' \)

8: \( X_{V'} \leftarrow \text{sort}(X_{V'}, V' \cap C[j]) \) \( \triangleright \) Sort value combinations in \( X_{V'} \) on the attributes that are common with \( C[j] \)

9: \( X_{V'}, X_{C_j} \leftarrow \text{ALIGN}(X_{V'}, X_{C_j}) \) \( \triangleright \) Align \( X_{V'} \) and \( X_{C_j} \)

10: \( V' \leftarrow V' \cup C[j] \) \( \triangleright \) Add attributes of \( C[j] \) in \( V' \)

11: \( X_{V'} \leftarrow \text{MERGE}(X_{V'}, X_{C_j}) \) \( \triangleright \) Merge the two sets \( X_{V'} \) and \( X_{C_j} \)

12: **end for**

13: \( X_V \leftarrow X_{V'} \) \( \triangleright \) Assign \( X_{V'} \) to \( X_V \) which is the desired output
Algorithm 3 Order Cliques Procedure

Input: \( \{c_i\}_{i \in [l]}, G \) \hspace{1cm} \triangleright \text{Set of cliques and the graph } G \\
Output: \( C \) \hspace{1cm} \triangleright \text{Ordered set of cliques}

Functionalities Used:

- \( \textit{findAllPaths}(P, Q) \) \hspace{1cm} \triangleright \text{Returns all paths between two sets of vertices } P \text{ and } Q \\
- \( \textit{getAdjacentCliques}(C) \) \hspace{1cm} \triangleright \text{Returns the set of cliques that share a vertex with any clique in the set of cliques } C \\

1: \textbf{Init:} \( C \leftarrow c_1, \ \textit{visited} \leftarrow c_1, \ k \leftarrow 1 \) \\
2: \textbf{while } k \neq l \textbf{ do} \hspace{1cm} \triangleright \text{Start with the first clique} \\
3: \hspace{1cm} \textbf{for } c_j \text{ in } \textit{getAdjacentCliques}(C) \textbf{ do} \\
4: \hspace{1cm} \hspace{1cm} \textit{flag} \leftarrow \textit{true} \\
5: \hspace{1cm} \hspace{1cm} \textit{commonVertices} \leftarrow \textit{visited} \cap c_j \hspace{1cm} \triangleright \text{get the common vertices between the adjacent clique } c_j \text{ and the observed set of cliques} \\
6: \hspace{1cm} \hspace{1cm} \textbf{for } \textit{path} \text{ in } \textit{findAllPaths} (\textit{visited}, c_j) \textbf{ do} \hspace{1cm} \triangleright \text{Find all paths for each vertex in } c_j \text{ to any vertex of the observed set of cliques. Choose } c_j \text{ if all the paths contain at least one common vertex} \\
7: \hspace{1cm} \hspace{1cm} \hspace{1cm} \textbf{if } \textit{path} \cap \textit{commonVertices} = \emptyset \textbf{ then} \\
8: \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \textit{flag} \leftarrow \textit{false} \\
9: \hspace{1cm} \hspace{1cm} \hspace{1cm} \textbf{break} \\
10: \hspace{1cm} \hspace{1cm} \textbf{end if} \\
11: \hspace{1cm} \hspace{1cm} \textbf{end for} \\
12: \hspace{1cm} \hspace{1cm} \textbf{if } \textit{flag} \textbf{ then} \\
13: \hspace{1cm} \hspace{1cm} \hspace{1cm} \textit{visited} \leftarrow \textit{visited} \cup c_j \\
14: \hspace{1cm} \hspace{1cm} \hspace{1cm} \textit{C}[k] \leftarrow c_j \\
15: \hspace{1cm} \hspace{1cm} \hspace{1cm} k \leftarrow k + 1 \\
16: \hspace{1cm} \hspace{1cm} \textbf{break} \\
17: \hspace{1cm} \hspace{1cm} \textbf{end if} \\
18: \hspace{1cm} \hspace{1cm} \textbf{end for} \\
19: \hspace{1cm} \textbf{end while} \\
20: \textbf{return } C
Algorithm 4 Align Procedure

**Input:** \( \{X_i : (x_i, k_i)\} \) \( _{i \in m} \), \( \{Y_j : (y_j, l_j)\} \) \( _{j \in n} \) \( \triangleright \) Set of value combinations with cardinalities for two sets of size \( m \) and \( n \) respectively

**Output:** \( X, Y \) \( \triangleright \) Returns the two sets after alignment

**Functionalities Used:**
- \( \text{split}(Z, r, \text{value}) \) \( \triangleright \) Returns the set after splitting its \( r^{th} \) entry \( (z_r, \text{count}_r) \) into \( Z_r : (z_r, \text{value}) \) and \( Z_{r+1} : (z_r, \text{count}_r - \text{value}) \)

1: **Init:** \( i \leftarrow 0, j \leftarrow 0 \)
2: **while** \( X_i \) exists **do** \( \triangleright \) Split \( X_i \) or \( Y_j \) depending on whose corresponding cardinality is greater
3: \hspace{1em} **if** \( k_i < l_j \) **then**
4: \hspace{2em} \( \text{split}(Y, j, k_i) \)
5: \hspace{1em} **else**
6: \hspace{2em} **if** \( k_i > l_j \) **then**
7: \hspace{3em} \( \text{split}(X, i, k_j) \)
8: \hspace{2em} **end if**
9: **end if**
10: \( i \leftarrow i + 1 \)
11: \( j \leftarrow j + 1 \)
12: **end while**
13: **return** \( X, Y \)

Algorithm 5 Merge Procedure

**Input:** \( \{X_i : (x_i, k_i)\} \), \( \{Y_i : (y_i, k_i)\} i \in [n] \) \( \triangleright \) Set of value combinations with cardinalities for two sets of same sizes

**Output:** \( Z \) \( \triangleright \) Merged Set

1: **Init:** \( Z \leftarrow \emptyset \) \( \triangleright \) Initialize the merged set with an empty set
2: **for** \( i \leftarrow 1 \) to \( n \) **do**
3: \hspace{1em} \( z_i \leftarrow x_i \cup y_i \) \( \triangleright i^{th} \) row of \( z \) is simply the join of \( x_i \) and \( y_i \)
4: \hspace{1em} \( Z_i \leftarrow (z_i, k_i) \) \( \triangleright \) cardinality of the \( i^{th} \) row of \( z \) is same as that of \( i^{th} \) row of \( X \) (or \( Y \))
5: **end for**
6: **return** \( Z \)
3.2.3.2 Making Views Consistent

So, after using the Clique Merging Algorithm, we obtain the solutions for every view. A solution for a view $V_i$ is of the form:

$$\{(x, k_x)\}_{x \in \text{Dom}(V_i)}$$

These solutions might be inconsistent as these independently solved views might not be able to give a valid solution for the relations. The relations have referential constraints that might get violated. For example, let there be two relations $R_i$ and $R_j$ such that $R_i$ depends on $R_j$. In this case, for all $x_i$ where $x_i \in \text{Dom}(V_i)$, when we project the value combination corresponding to the attributes of $V_j$, then the resultant value combination should be present in the solution-set of $V_j$. If this condition is violated, then the views are said to be inconsistent.

To make it consistent, we need to add extra tuples in solution set of $V_j$ such that the above condition is satisfied. Note that such an inconsistency will arise because the LP solver might give us a solution that does not obey consistency across views. If the solution has a case where the variable corresponding to an interval combination of $V_j$ gets a value 0 and the corresponding variable in $V_i$ (the variable having the same interval combination for the attributes of $V_i$) gets a non-zero value, then this inconsistency will arise.

The schematic constraints (like referential integrity) are extremely important when dealing with database systems. We can afford some errors in conforming to the statistical constraints to satisfy the schematic constraints. Therefore, to ensure that the final relations have referential integrity, or simply to make the view solutions consistent, the technique used will induce some errors in satisfying the cardinality constraints. However, in our experiments we show that these errors are minor.

To make the views consistent, we use the View Consistency Algorithm (Algorithm 6). We first run a topological sort on the dependency graph. Since we assumed that the dependencies are non-cyclic, we are guaranteed to get an ordering of views. Now, for each view $V_i$ in the order, if there exists a value combination $x_{i-1}$ in the solution set of $V_{i-1}$, for which if we project the value combination corresponding to the $V_i$'s attributes, this projected solution (say $x'_{i-1}$) does not exist in solution set of $V_i$. If so, we add another entry $(x'_{i-1}, 1)$ in the solution set of $V_i$. 


Algorithm 6 View Consistency Algorithm

Input: $\{X_i : \{(x, k_x)\}_{x \in \text{Dom}(V_i)}\}_{i \in [n]}$, Dependency Graph $G$ \hspace{1cm} $\triangleright$ Solution set for each view

Output: $\mathbb{Y}$ \hspace{1cm} $\triangleright$ Consistent solution set for each view

Functionalties Used:

- $\text{topologalsort}(G)$ \hspace{1cm} $\triangleright$ Returns a view tranveral order by taking the dependency graph as the input
- $\text{project}(X_i, V_j)$ \hspace{1cm} $\triangleright$ Returns the value combinations in $X_i$ corresponding to the attributes present in $V_j$

1: Init: $\mathbb{Y}^1 \leftarrow X^1$

2: for $i \leftarrow 2$ to $n$ do

3: $\text{tempset} \leftarrow \text{project}(X_i^{i-1}, V_i) - X^i(x)$

4: if $\text{tempset} \neq \emptyset$ then

5: for $z$ in $\text{tempset}$ do

6: $\mathbb{Y}^i \leftarrow \mathbb{Y}^i \cup (z, 1)$

7: end for

8: end if

9: end for

10: return $\mathbb{Y}$

3.2.3.3 Constructing Relation Summary

Once we have the consistent views solutions, we next need to get the relation summaries from them. For this, we create a summarized relation set $\widetilde{R}_i$ for each relation $R_i$. This set consists of the attributes in $R_i$ except the primary key attribute. In addition, for each entry in $\widetilde{R}_i$, we maintain the corresponding number of tuples (like we have it in view solutions). For the attributes that are common between the summarized relation set and the corresponding view solution set, the value combinations and corresponding cardinalities are directly borrowed.

What remains are the foreign key attributes. For filling a foreign key attribute $fk$, we first need to see the view corresponding to the relation that the foreign key refers to. Say the view thus obtained is $V_j$. Now, to fill the $fk$ value in $r^{th}$ row of $\widetilde{R}_i$, we see the value combination in the $r^{th}$ row of view solution set of $V_i$. From this value combination, we project the attributes of $V_j$. Say the combination thus obtained is $v$. Now, we iterate over the solution set of $V_j$ and keep summing up all the cardinality entries seen until we find $v$. This summed value gives us the $fk$ value corresponding to $r^{th}$ row of $\widetilde{R}_i$.

We thus obtain the set $\widetilde{R}_i$ for each relation $R_i$. These are the relation summaries that are
sufficient to generate tuples on demand.

### 3.2.4 Tuple Generator

The relation generator gives us the summaries for each relation. An entry in this set is of the form \((x, k_x)\), where \(k_x\) represents the number of tuples having \(x\) value combination. We consider the \(PK\) values to be the row numbers of the relation. Therefore, to get the \(r^{th}\) tuple of relation \(\mathcal{R}_i\), the \(PK\) is chosen as \(r\) and the rest of the attributes come from the summarized relation. We iterate over the rows of \(\tilde{\mathcal{R}}_i\) and keep summing the cardinalities until the summation becomes greater than \(r\). Say the summation crosses the value \(r\) in \(j^{th}\) row of \(\tilde{\mathcal{R}}_i\). So the rest of the values of the \(r^{th}\) tuple of \(\mathcal{R}_i\) are precisely the one that are present in the \(j^{th}\) row of \(\tilde{\mathcal{R}}_i\).

### 3.3 Projections

In this section, we propose an algorithm for handling cardinality constraints having projection operators. Note that, the constraints involving duplicate-preserving projections do not affect the data generation technique because the input and output cardinalities of a duplicate-preserving projection operator are identical. Therefore, we assume that the cardinality constraints have only duplicate-eliminating projections. Solving the problem over a single table with constraints that have just projections and no selections is non-trivial and has connections to known hard problems in combinatorial geometry [4]. The solution that we propose depends on the size of the database and therefore is applicable for small scale projections only.

Let us assume that the constraints that include projections are of the form:

\[
|\pi_A(\mathcal{R}_{i_1} \bowtie \ldots \bowtie \mathcal{R}_{i_p})| = k
\]

The joins can be removed by converting them to views as we did before. So now we get constraints of the form:

\[
|\pi_A(\mathcal{V}_i)| = k
\]

(3.11)

Note that since filter predicate will only limit the domain of the attributes and can easily fit in our model, we have omitted them for simplicity.

The algorithm has the following steps:

- Construct a graph \(G\) with nodes corresponding to the attributes of the view. We add an edge between nodes of the attributes \((A_i, A_j)\) if there exists a constraint \(C\) of the form (6.1) where \(A_i, A_j \in A\).
- Find all connected components in \(G\).
• For each connected component, we solve an ILP that is formulated according to the algorithm that we present next.

3.3.1 ILP Formulation For Projections

Let the connected component have the attributes $A_1, A_2, \ldots, A_N$ and $r$ be the number of rows that are to be generated.

For now, we are considering domain for all attribute to be $\{1, 2, \ldots, D\}$. We can reduce the size of the domain of an attribute $A_i$ to the number of distinct values in $A_i$. Obtaining this value is not difficult.

We first define some notations. Let $\mathcal{R}$ be the relation to be generated and $\mathcal{R}_j$ denotes the $j^{th}$ row in $\mathcal{R}$. Now, let us define an indicator variable $y_{k_1, \ldots, k_N}^j$ as

$$y_{k_1, \ldots, k_N}^j = \begin{cases} 1 & \text{if } \mathcal{R}_j = (k_1, \ldots, k_N), \\ 0 & \text{otherwise}. \end{cases}$$

signifying if $j^{th}$ row is $(k_1, \ldots, k_N)$. Here $(k_1, \ldots, k_N) \in \mathcal{D} = \text{Dom}(A_1) \times \text{Dom}(A_2) \times \ldots \times \text{Dom}(A_N)$.

Further, let us define another indicator variable $\tilde{Y}_{p_1, \ldots, p_N}$ as

$$\tilde{Y}_{p_1, \ldots, p_N} = \begin{cases} 1 & \sum_{j=1}^r y_{p_1, \ldots, p_N}^j > 0 \\ 0 & \text{otherwise} \end{cases}$$

Here, $p_i \in \text{Dom}(A_i) \cup \{\ast\}$. Here $\ast$ indicates any value from the domain. The variable $\tilde{Y}_{p_1, \ldots, p_N}$ indicates the presence of a tuple of the form $p_1, \ldots, p_N$ in the database.

Now, for a constraint $C$ of the form as mentioned in (6.1), let $\mathcal{P}_C = \{p_i, \ldots, p_N\}$ where $p_i = '\ast'$, if $A_i \notin \mathcal{A}$. The constraint can then be expressed as:

$$\sum_{p_i \forall A_i \in \mathcal{A}} \tilde{Y}_{p_C} = k$$

The ILP can be formulated as:

$$\max \sum_{C} \tilde{Y}_{p_C}$$

such that, $\forall C$, we add

$$\sum_{p_i \forall A_i \in \mathcal{A}} \tilde{Y}_{p_C} = k$$

(3.12)
and
\[ 0 \leq Y_{pe} \leq 1 \] (3.13)

Also, we add the following two set of equations \( \forall j \in [r] \):
\[
\sum_{k_1, \ldots, k_n \in D} y^j_{k_1, \ldots, k_n} = 1
\]
(3.14)

and
\[ 0 \leq y^j_{k_1, \ldots, k_n} \leq 1 \] (3.15)

Finally we add the condition:
\[
\tilde{Y}_{p_1, \ldots, p_N} \leq \sum_{j=1}^{r} y^j_{p_1, \ldots, p_N}
\]
(3.16)

3.4 Metadata Constraints

So far we looked at the constraints that are taken from the ALQPs. The algorithm for data
generation also supports constraints that are generated from the statistical metadata. The
statistical metadata consists of \textit{one-dimensional histograms} corresponding to all the attributes
across various relations in the database. Enforcing these constraints guarantees that the gen-
erated database also conforms to the metadata of the original database.

The histograms can be expressed in the form of cardinality constraints (as defined in Equation
3.1) by adding one constraint for each histogram bucket. The constraint corresponding to a
bucket with boundaries \([l, h]\) (for an attribute \(A\) in relation \(R\)) having \(k\) tuples can be written as
\[ |\sigma_{l \leq A \leq h}(R)| = k \]

Likewise, all the histogram buckets can be converted to an equivalent cardinality constraint.
The constraints can be supplied to the LP formulator.
Chapter 4

Experiments

4.1 Setup

For experiments, we took an instance of TPC-DS benchmark database of size 10 GB and a workload of 14 TPC-DS queries that were modified to suit our assumptions. The resultant workload had simple select - join queries that can be written easily in the form as expressed in equation (3.1).

Since the algorithm depends on the cardinality constraints and not the explicit values in the database, working on TPC-DS is reasonable. Working on realistic databases would add no extra complexity to the algorithm.

The objective of the experiment is to construct a synthetic database that can ensure similar volumetric processing of data for each query in the workload as it is on the original TPC-DS database instance. For this, the workload of 14 queries was first executed on the original TPC-DS database and the corresponding ALQP for each of the queries was fetched. We gave these 14 ALQPs along with the database schema as the input to the generator. And we got a corresponding synthetic database. We then again executed this workload on synthetic database.

4.2 Results

We found that the synthetic data indeed satisfied the cardinality constraints except for the small additive errors that were present due to the additional tuples resulting from View Consistency Algorithm. We report the sizes of each of the base tables in Table 4.1. We can see that the difference is minor. The difference of cardinalities for any intermediate operator is upper bounded by the cardinality difference in corresponding base tables.

Figure 4.1 shows the complete execution tree for the following query:
<table>
<thead>
<tr>
<th>Table Name</th>
<th>Original DB</th>
<th>Synthetic DB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
<td>102000</td>
<td>102006</td>
</tr>
<tr>
<td>Store</td>
<td>102</td>
<td>103</td>
</tr>
<tr>
<td>Catalog_sales</td>
<td>14401261</td>
<td>14401263</td>
</tr>
<tr>
<td>Customer_address</td>
<td>250000</td>
<td>250001</td>
</tr>
<tr>
<td>Customer_demographics</td>
<td>1920800</td>
<td>1920802</td>
</tr>
<tr>
<td>Inventory</td>
<td>133110000</td>
<td>133110000</td>
</tr>
<tr>
<td>Warehouse</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Customer</td>
<td>500000</td>
<td>500001</td>
</tr>
<tr>
<td>Promotion</td>
<td>500</td>
<td>501</td>
</tr>
<tr>
<td>Date_dim</td>
<td>73049</td>
<td>73066</td>
</tr>
<tr>
<td>Store_sales</td>
<td>28800991</td>
<td>28800992</td>
</tr>
<tr>
<td>Web_sales</td>
<td>7197559</td>
<td>7197566</td>
</tr>
</tbody>
</table>

Table 4.1: Cardinalities of Base Tables

```sql
SELECT *
FROM SS, CD, D, S, I
WHERE ss_sold_date_sk = d_date_sk
AND ss_item_sk = i_item_sk
AND ss_store_sk = s_store_sk
AND ss_cdemo_sk = cd_cdemo_sk
AND cd_gender = 'F'
AND cd_marital_status = 'U'
AND cd_education_status = 'Secondary'
AND d_year = 1999
AND s_state = 'TN'
```
We can see that the execution cardinalities on the original and synthetic databases are very close to each other.

Further, we also wanted to check the impact of the optimizations in reducing the number of variables. The effect of domain decomposition is straightforward as it reduced the effective domain sizes of each attribute exponentially. To check the impact of View Decomposition Algorithm, we ran our LP Formulator with and without the optimization. Table 4.2 shows the number of variables for these two cases.

<table>
<thead>
<tr>
<th>View Name</th>
<th>Original</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
<td>105</td>
<td>15</td>
</tr>
<tr>
<td>Store</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Catalog_sales</td>
<td>324</td>
<td>66</td>
</tr>
<tr>
<td>Date_dim</td>
<td>864</td>
<td>49</td>
</tr>
<tr>
<td>Store_sales</td>
<td>437400</td>
<td>538</td>
</tr>
<tr>
<td>Web_sales</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 4.2: Number of variables
Chapter 5

Conclusion and Future Work

We considered the problem of execution-time testing for database systems. We developed a data generator that is capable of mimicking real customer scenarios for a predefined workload. Our generation ensures that the data stored on the disk is independent of the size of the database. This feature enables us to support big data scenarios as well. Further, our generation algorithm is capable of supplying tuples on demand. Due to this, we adhere to CODD’s philosophy of creating a database with no static data.

Currently, our implementation supports only simplified queries that contain only select and join operators. Our next target is to optimize our algorithm for handling large scale projections and implement it. Operators like Group By and Union also depend on projection. If we can handle projections efficiently, handling these operators is also straightforward. Also, we propose handling generic schemas and joins as a future work.
Bibliography


[16] Rakshit S. Trivedi, I. Nilavalagan, and Jayant R. Haritsa. CODD: COonstructing dataless databases. DBTest, 2012. 1, 2