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# LRU-K PAGE REPLACEMENT ALGORITHM

E0 261

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# ABOUT PAPER

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- Authors:
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- Facts:
  - Presented at ACM SIGMOD Conference May 1993, Washington D.C. and published in its Proceedings, May 1993.
  - Paper has 1235 Citations till date.
  - Implementation in C available on author's website.

# LRU

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- Concept:
  - When a new buffer page is needed, the buffer pool manager drops the page from buffer that has not been accessed for the longest time.
- Why LRU?
  - Most logical and intuitive
  - Easy to implement
  - Good chance of a question in GATE exam => Increased chances of getting into IISc!

# CASE - I

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- A multi-user database application, which references randomly chosen customer records through a clustered B-tree indexed key, CUST-ID, to retrieve desired information.
- If pattern of access is: I1, R1, I2, R2, I3, R3, ....
- In traditional LRU, we may drop the index pages, which would then be referenced again, resulting in increased I/O.

# CASE - II

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- Consider a multi-process database application with good “locality” of shared page reference.
- Suppose, if 5000 buffered pages out of 1 million disk pages get 95% of the references by concurrent processes.
- Sequential scans will replace commonly referenced pages in buffer with pages unlikely to be referenced again, a common complaint in many commercial situations!

# PROBLEM

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- LRU decides what page to drop from buffer based on too little information, limiting itself to only the time of last reference.
- LRU is unable to differentiate between pages that have relatively frequent references.
- Results in increased I/O.
- In general, bad for database applications!

# PRIOR LITERATURE

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- Solutions in the literature majorly fall into two categories :
- **Page pool tuning :**
- DBA constructs page pools, separating different reference patterns into different buffer pools.
- Supported by some commercial database systems, but **requires great manual effort!**

# PRIOR LITERATURE

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- Query Plan Analysis :
- Query optimizer should provide hints about the usage pattern of a query plan.
- In multi-user situations, query optimizer plans may **overlap** in complicated ways.
- There is a need for a more general and global page replacement policy!



# TODAY's PAPER

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- We'll see LRU-K algorithm!
- It discriminates well between page sets with different levels of reference frequency (e.g., index pages vs. data pages).
- It adapts itself to evolving access patterns.
- It is self-reliant in that it does not need any external hints.
- It is fairly simple and incurs little bookkeeping overhead.

# INDEPENDENT REFERENCE MODEL

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- The Independent Reference Model is a conceptual model used in the analysis of storage system like disk drives, caches, etc.
- Under this model the references to stored objects are independent random variables.



# INDEPENDENT REFERENCE MODEL

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- Set  $N = \{1, 2, \dots, n\}$  of disk pages.
- The database system under study makes a succession of references to these pages specified by the reference string:  $r_1, r_2, \dots, r_t$  where  $r_t = \text{page } p$  ( $p \in N$ )
- At instant  $t$ , we assume that each disk page  $p$  has a well defined probability,  $b_p$ , to be the next page referenced.

$$\text{Prob}(r_{t+1} = p) = b_p \quad \forall p \in N.$$

- We assume that  $b_p$  is independent of  $t$ .

# INDEPENDENT REFERENCE MODEL

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- Each disk page  $p$  has an **expected reference interarrival time**,  $I_p$ , the time between successive occurrences of  $p$
- For the reference string in last slide, we have

$$I_p = \frac{1}{b_p}$$

- **GOAL** : The system then attempts to keep in memory buffers only those pages with **shortest access interarrival times**, or equivalently **greatest probability of reference**.

# BACKWARD K-DISTANCE

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Given a reference string known up to time  $t$ ,  $r_1, r_2, \dots, r_t$ , the backward K-distance  $b_t(p, K)$  is the distance backward to the  $K^{\text{th}}$  most recent reference to the page  $p$ :

$$\begin{aligned} b_t(p, K) &= x, && \text{if } r_{t-x} \text{ has the value } p \text{ and there have been} \\ &&& \text{exactly } K-1 \text{ other values } i \text{ with} \\ &&& t-x < i \leq t, \text{ where } r_i = p, \\ &= \infty, && \text{if } p \text{ does not appear at least } K \text{ times in} \\ &&& r_1, r_2, \dots, r_t \end{aligned}$$

- The victim page (page to be dropped) is the one whose **Backward K-distance is the maximum** of all pages in buffer.
- Conflict : Incase of  $>1$  pages with  $b_t(p, K) = +\infty$ , follow classical LRU to choose victim.

# EXAMPLE (K=2)

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- Consider the reference string :

7, 5, 6, 2, 9, 1, 8, 7, 6, 8, 1  
t : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10  
x :  $\infty$ ,  $\infty$ ,  $\infty$ ,  $\infty$ ,  $\infty$ ,  $\infty$ ,  $\infty$ , 7, 6, 4, 5

- And if we have LRU-2,
  - $b_t(p=1, k=2) \Rightarrow 5$
  - $b_t(p=6, k=2) \Rightarrow 6$
  - $b_t(p=9, k=2) \Rightarrow +\infty$

# BACKWARD K-DISTANCE

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- Ability to make decisions by measurement of an actual interarrival between references, rather than estimating simply by a lower bound of the time back to the most recent reference!

But, we still have problems :

- Early page replacement
- Retained information problem

# EARLY PAGE REPLACEMENT

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- Four ways in which a pair of references might end up to same disk page :
    - (1) Intra-Transaction
    - (2) Transaction-Retry
    - (3) Intra-Process
    - (4) Inter-Process
- (1),(2),(3) are called  
==> **correlated reference pairs**  
comparatively short span
- ==> comparatively longer span



# EARLY PAGE REPLACEMENT

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- Interarrival time should be calculated based on non-correlated access pairs!
- Each successive access by the same process within a **time-out period (defined)** is assumed to be correlated.
- Refer to the approach, which associates correlated references, as the **Time-Out Correlation method**.
- Refer to the time-out period as the **Correlated Reference Period**.

# RETAINED INFO PROBLEM

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- Each time a page  $p$  is referenced, it is made buffer resident.
- If our estimate of  $b_t(p, K) = +\infty$ , we drop it!  
But what if the page is popular, but just not able to meet the backward- $k$  criteria?
- Though the page is frequently referenced, we would have no history about it to recognize this fact!
- We need to **maintain history information** about any page for some period after its most recent access. We refer to this period as the **Retained Information Period**.

# DATA STRUCTURES : LRU-K

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- **HIST(p)**
  - Denotes the history control block of page p
  - Contains the times of the K most recent references to page p
  - HIST(p,1), HIST(p,2) denotes the time of last and second last references to page p.
- **LAST(p)**
  - Denotes the time of the most recent reference to page p.
- Maintained IN-MEMORY! Why not store in page header?

# PSEUDO CODE : LRU-K

Procedure to be invoked upon reference to page p at time t:

```
if p is already in the buffer
then /* update history information of p */
  if t - LAST(p) > Correlated_Reference_Period
  then /* a new, uncorrelated reference */
    correl_period_of_refd_page := LAST(p) - HIST(p,1)
    for i := 2 to K do
      HIST(p,i) := HIST(p,i-1) +
                    correl_period_of_refd_page
    od
    HIST(p,1) := t
    LAST(p) := t
  else /* a correlated reference */
    LAST(p) := t
  fi
else /* select replacement victim */
  min := t
  for all pages q in the buffer do
    if t - LAST(q) > Correlated_Reference_Period
      /* if eligible for replacement */
      and HIST(q,K) < min
      /* and max Backward K-distance so far */
```

```
then
  victim := q
  min := HIST(q,K)
fi
od
if victim is dirty then
  write victim back into the database fi
/* now fetch the referenced page */
fetch p into the buffer frame previously held by victim
if HIST(p) does not exist
then /* initialize history control block */
  allocate HIST(p)
  for i := 2 to K do HIST(p,i) := 0 od
else
  for i := 2 to K do HIST(p,i) := HIST(p,i-1) od
fi
HIST(p,1) := t
LAST(p) := t
fi
```

# OPTIMALITY OF LRU-K

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- Set  $N = \{1, 2, \dots, n\}$  of disk pages.
- Set  $M = \{1, 2, \dots, m\}$  of memory buffers.  $1 \leq m \leq n$
- Reference string:  $r_1, r_2, \dots, r_t$  where  $r_t = \text{page } p$  ( $p \in N$ )
- Recall that by Independence reference model,

Mean interarrival time :  $I_p = \frac{1}{b_p}$

# Ao ALGORITHM

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- Let Ao denote the buffering algorithm that replaces the buffered page  $p$  in memory whose expected value  $l_p$  is the maximum, i.e., the page for which  $b_p$  is smallest.
- Algorithm Ao is optimal under the independent reference assumption. (Proof in the full version of the paper)
- The best possible approximation of Ao is the LRU-K algorithm. As  $K$  increases, LRU-K gives very close results to the optimal algorithm.

# TWO-POOL EXPERIMENT

- Pool 1 with  $N_1$  pages and Pool 2 with  $N_2$  pages
- In this two pool experiment, alternating references are made :  
 $I_1, R_1, I_2, R_2, I_3, R_3, \dots$
- (Recall that this experiment models Case-I in the initial slides!)

| B   | LRU-1 | LRU-2 | LRU-3 | $A_0$ | $B(1)/B(2)$ |
|-----|-------|-------|-------|-------|-------------|
| 60  | 0.14  | 0.291 | 0.300 | 0.300 | 2.3         |
| 80  | 0.18  | 0.382 | 0.400 | 0.400 | 2.6         |
| 100 | 0.22  | 0.459 | 0.495 | 0.500 | 3.0         |
| 120 | 0.26  | 0.496 | 0.501 | 0.501 | 3.3         |
| 140 | 0.29  | 0.502 | 0.502 | 0.502 | 3.2         |
| 160 | 0.32  | 0.503 | 0.503 | 0.503 | 2.8         |
| 180 | 0.34  | 0.504 | 0.504 | 0.504 | 2.5         |
| 200 | 0.37  | 0.505 | 0.505 | 0.505 | 2.3         |
| 250 | 0.42  | 0.508 | 0.508 | 0.508 | 2.2         |
| 300 | 0.45  | 0.510 | 0.510 | 0.510 | 2.0         |
| 350 | 0.48  | 0.513 | 0.513 | 0.513 | 1.9         |
| 400 | 0.49  | 0.515 | 0.515 | 0.515 | 1.9         |
| 450 | 0.50  | 0.517 | 0.518 | 0.518 | 1.8         |

Table 4.1. Simulation results of the two pool experiment, with disk page pools of  $N_1 = 100$  pages and  $N_2 = 10,000$  pages. The first column shows the buffer size  $B$ . The second through fifth columns show the hit ratios of LRU-1, LRU-2, LRU-3, and  $A_0$ . The last column shows the equi-effective buffer size ratio  $B(1)/B(2)$  of LRU-1 vs. LRU-2.

# ZIPFIAN RANDOM ACCESS

- A single pool of pages with skewed random access. We generated references to  $N$  pages with a Zipfian distribution of reference frequencies.

| B   | LRU-1 | LRU-2 | $A_0$ | $B(1)/B(2)$ |
|-----|-------|-------|-------|-------------|
| 40  | 0.53  | 0.61  | 0.640 | 2.0         |
| 60  | 0.57  | 0.65  | 0.677 | 2.2         |
| 80  | 0.61  | 0.67  | 0.705 | 2.1         |
| 100 | 0.63  | 0.68  | 0.727 | 1.6         |
| 120 | 0.64  | 0.71  | 0.745 | 1.5         |
| 140 | 0.67  | 0.72  | 0.761 | 1.4         |
| 160 | 0.70  | 0.74  | 0.776 | 1.5         |
| 180 | 0.71  | 0.73  | 0.788 | 1.2         |
| 200 | 0.72  | 0.76  | 0.825 | 1.3         |
| 300 | 0.78  | 0.80  | 0.846 | 1.1         |
| 500 | 0.87  | 0.87  | 0.908 | 1.0         |

Table 4.2. Simulation results on buffer cache hit ratios for random access with Zipfian 80-20 distribution to a disk page pool of  $N=1000$  pages.



# OLTP TRACE

- One-hour page reference trace of the production OLTP system of a large bank.
- This trace contained approximately 470,000 page references to a CODASYL database with a total size of 20 Gigabytes.

| B    | LRU-1 | LRU-2 | LFU  | B(1)/B(2) |
|------|-------|-------|------|-----------|
| 100  | 0.005 | 0.07  | 0.07 | 4.5       |
| 200  | 0.01  | 0.15  | 0.11 | 3.25      |
| 300  | 0.02  | 0.20  | 0.15 | 3.0       |
| 400  | 0.06  | 0.23  | 0.17 | 2.75      |
| 500  | 0.09  | 0.24  | 0.19 | 2.4       |
| 600  | 0.13  | 0.25  | 0.20 | 2.16      |
| 800  | 0.18  | 0.28  | 0.23 | 1.9       |
| 1000 | 0.22  | 0.29  | 0.25 | 1.6       |
| 1200 | 0.24  | 0.31  | 0.27 | 1.66      |
| 1400 | 0.26  | 0.33  | 0.30 | 1.5       |
| 1600 | 0.29  | 0.34  | 0.31 | 1.5       |
| 2000 | 0.31  | 0.36  | 0.33 | 1.3       |
| 3000 | 0.38  | 0.40  | 0.39 | 1.1       |
| 5000 | 0.46  | 0.47  | 0.44 | 1.05      |

Table 4.3. Simulation results on buffer cache hit ratios using an OLTP trace.

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# END LRU-K

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